Shifting Bottleneck Heuristic

Algorithm

1. Initialization

- $M_0 = \emptyset$ **(scheduled machines)**
- G **= only conjuctive arcs**
- $C_{\max}$ **= critical path in** G**.**

2. (Choice of machine.) For each $M_i \in M - M_0$**,**

- **generate the** $1|r_j|L_{\max}$ **schedule**
- **compute** $L_{|max}(i)$**.**

3. Scheduling the bottleneck machine

- **Let** k **be the machine that maximizes** $L_{\max}(i)$
- **Schedule** k **by the** $!|r_j|L_{\max}$ **solution**
- **Update** G
- $M_0 = M_0 \cup \{k\}$**.**

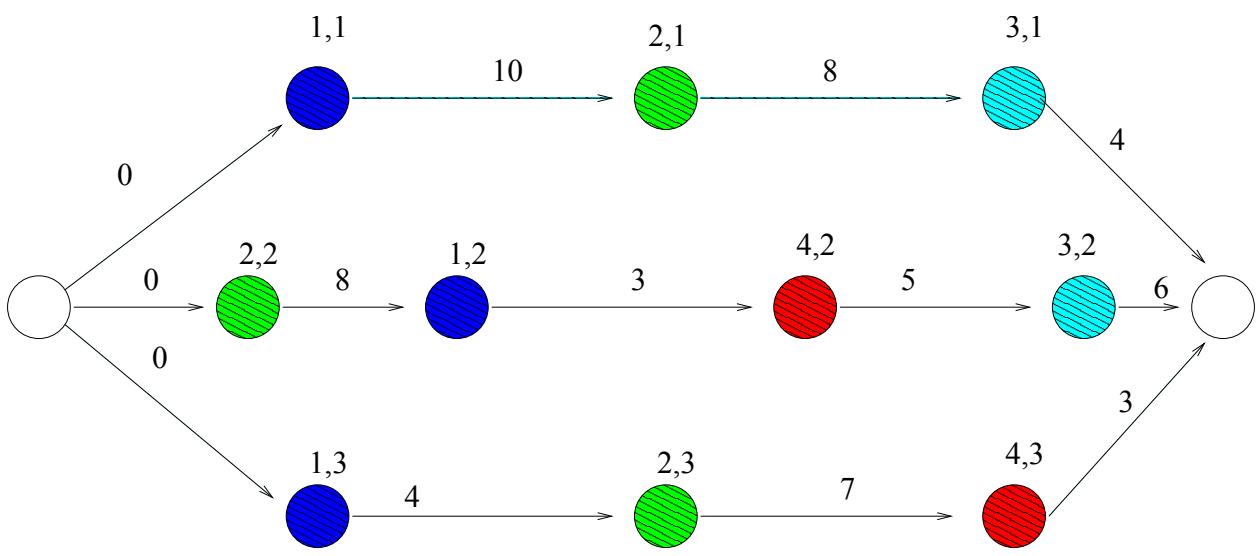
4. (Resequence already scheduled machines.) For each $M_i \in M_0 - \{k\}$

- **Delete disjunctive arcs for** M_i **from** G
- **Form the** $1|r_j|L_{\max}$
- **Reschedule** M_i **according to this schedule**

5. If $M = M_0$ **stop, else go to 2**

Example

jobs	machine sequence	processing times
1	**1,2,3**	$p_{11} = 10, p_{21} = 8, p_{31} = 4$
2	**2,1,4,3**	$p_{22} = 8, p_{12} = 3, p_{42} = 5, p_{32} = 6$
3	**1,2,4**	$p_{13} = 4, p_{23} = 7, p_{43} = 3$



critical path is 22

lower bound on C_{max} = 22

Goal: schedule w/ $C_{max} = 22$

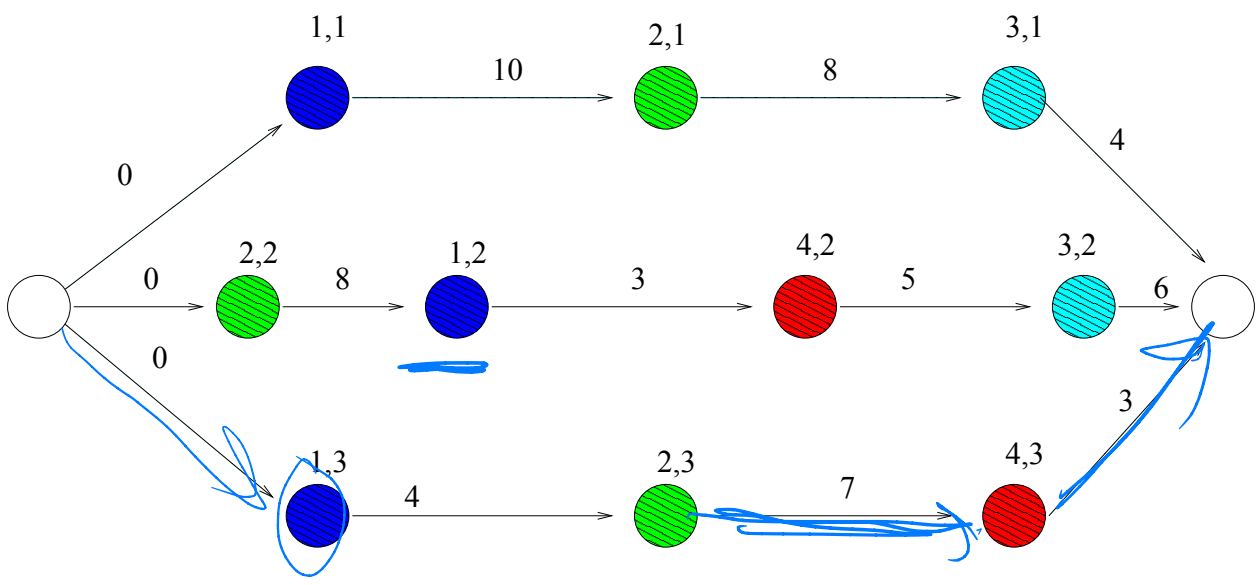
the only way we can possibly achieve $C_{max} = 22$ is to run job 1 on M_1 at time 0

Iteration 1

job 1

job 2

job 3



Form the $1|r_j|l_{\max}$ problems (Recall $d_j = LB - (CP - p_j)$)

Machine 1

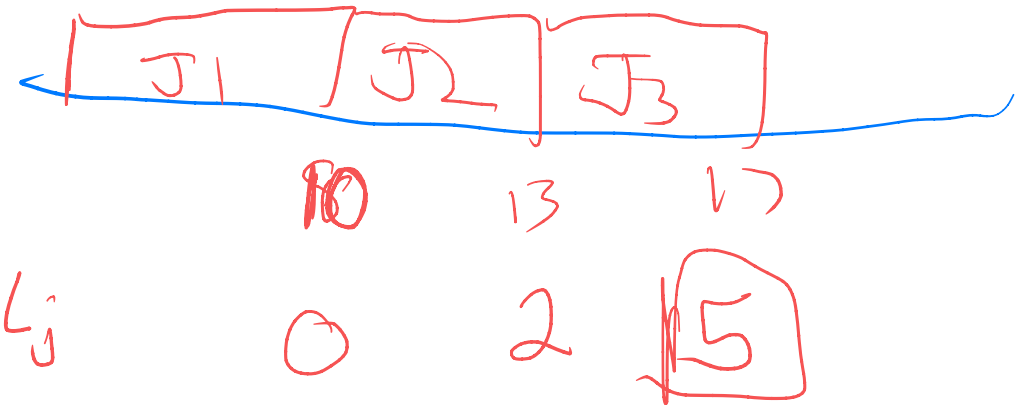
job	1	2	3
r_j	0	8	0
p_j	10	3	4
d_j	10	11	12

Optimal schedule 1,2,3, $L_{\max}(1) = 5$

22 − 12 = 10

22 − 11 = 11

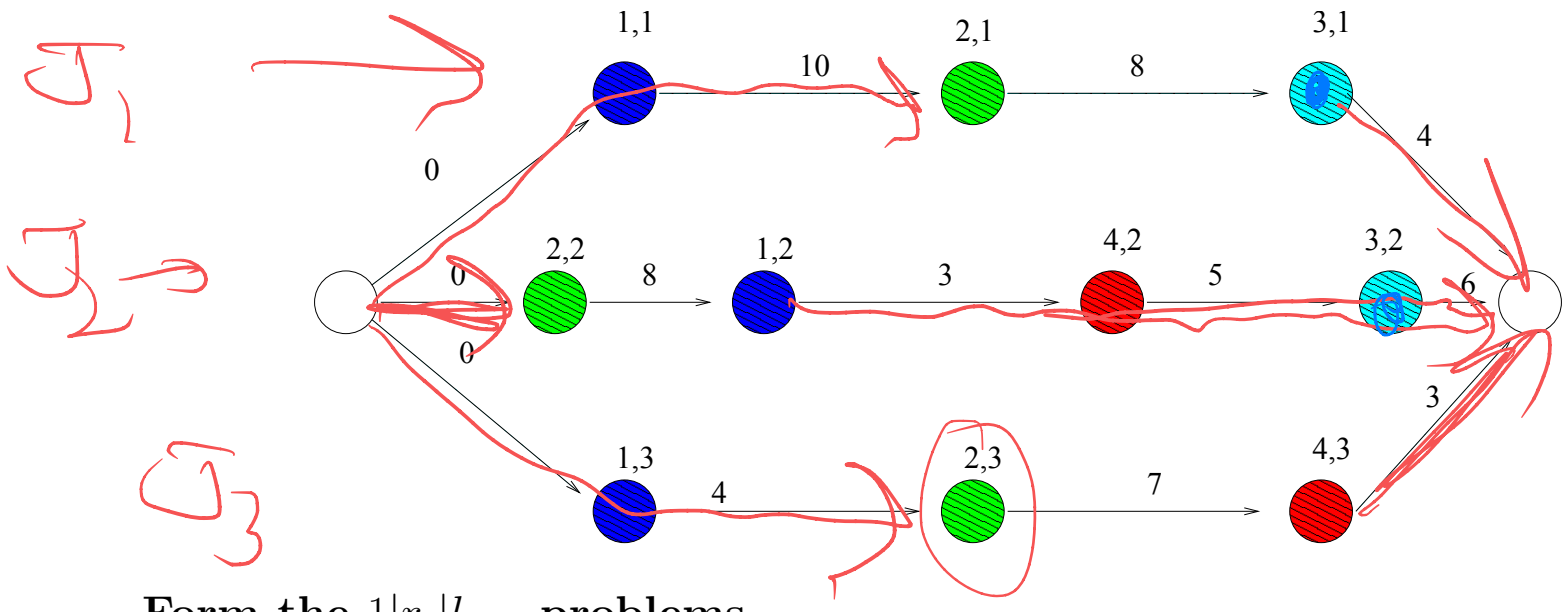
after job 3 finishes on M_1, job 3 still has 7+3=10 more units of processing ∴ job 3 must finish on M_1 by time 22 − 10 = 12



Cmax = 22

How can we achieve Cmax = 22 only looking at M_2

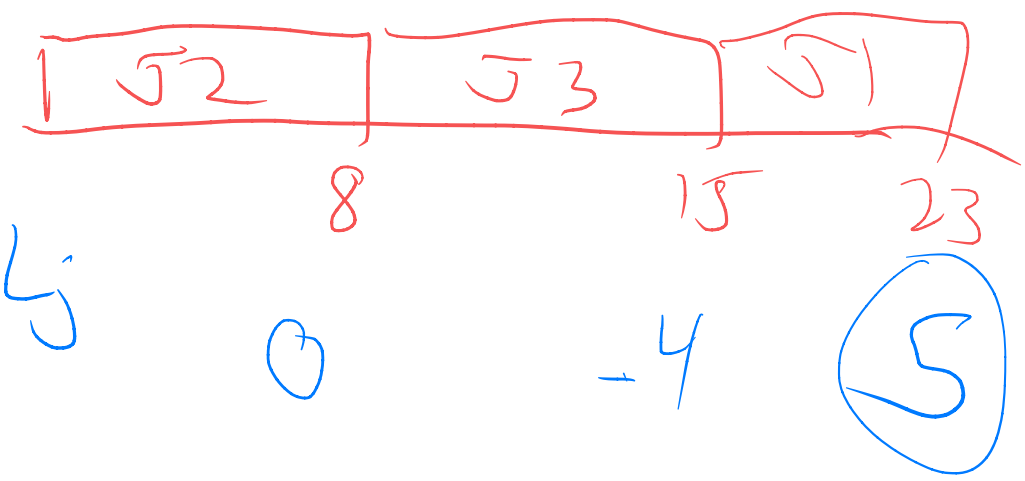
Iteration 1 (cont)



Form the $1|r_j|l_{\max}$ problems

Machine 2

job	1	2	3
r_j	10	0	4
p_j	8	8	7
d_j	18	8	19



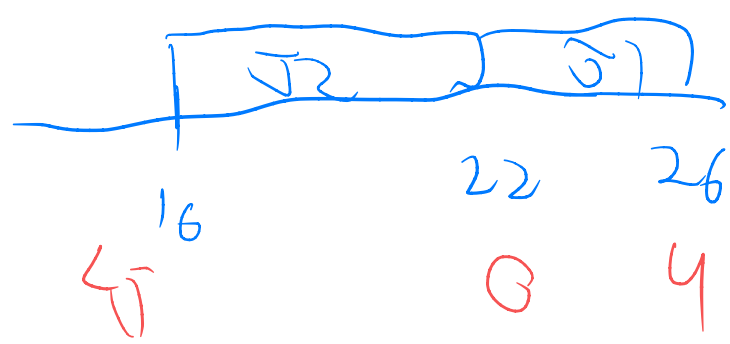
Optimal schedule **2,3,1** $L_{\max}(2) = 5$

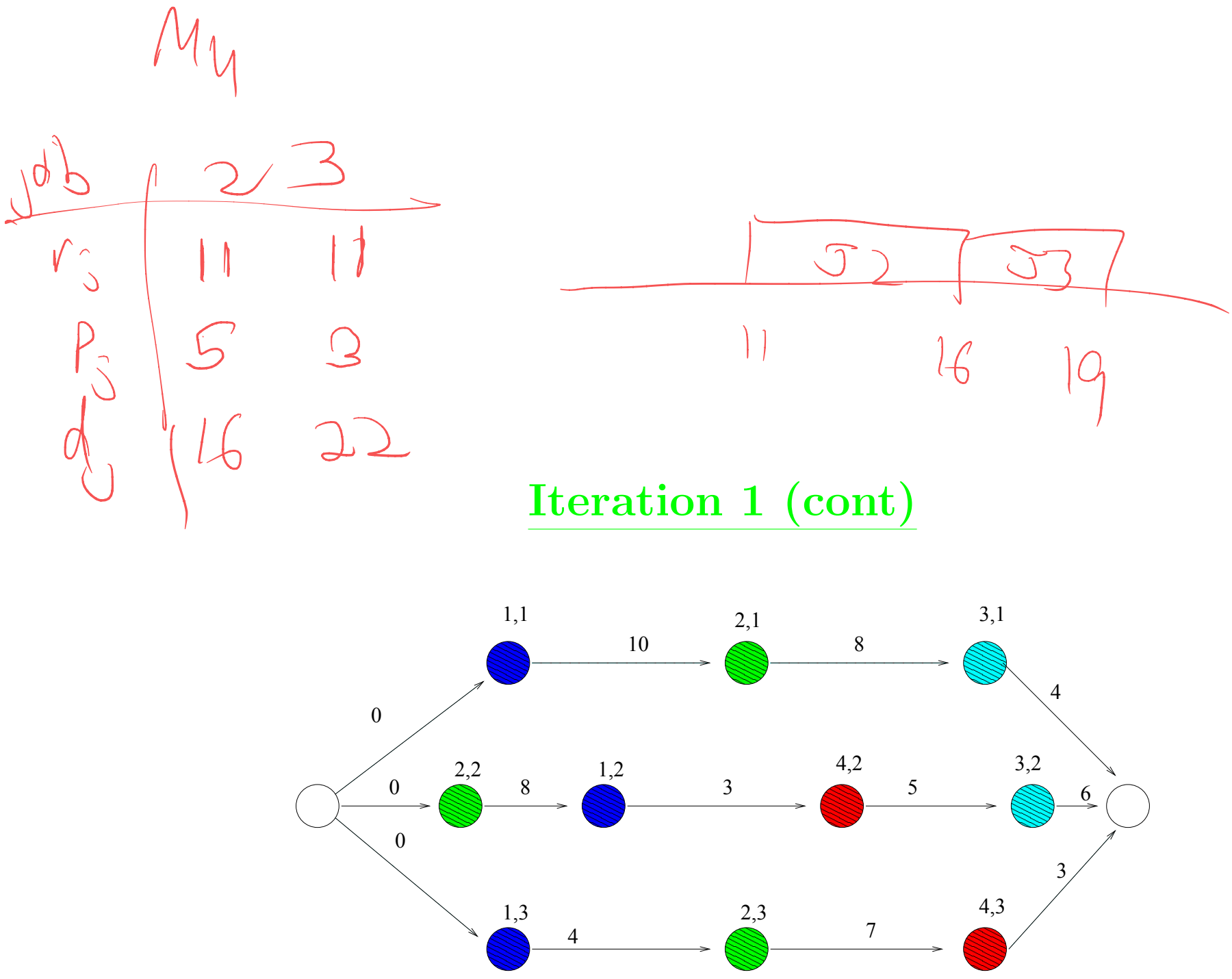
Similarly $L_{\max}(3) = 4$. $L_{\max}(4) = 0$.

Schedule M_1 **in the order 1,2,3.**

M_3

job	1	2
r_j	18	16
p_j	4	6
d_j	22	22





Iteration 1 (cont)

Form the $1|r_j|l_{\max}$ problems

Machine 2

job	**1**	**2**	**3**
r_j	**10**	**0**	**4**
p_j	**8**	**8**	**7**
d_j	**18**	**8**	**19**

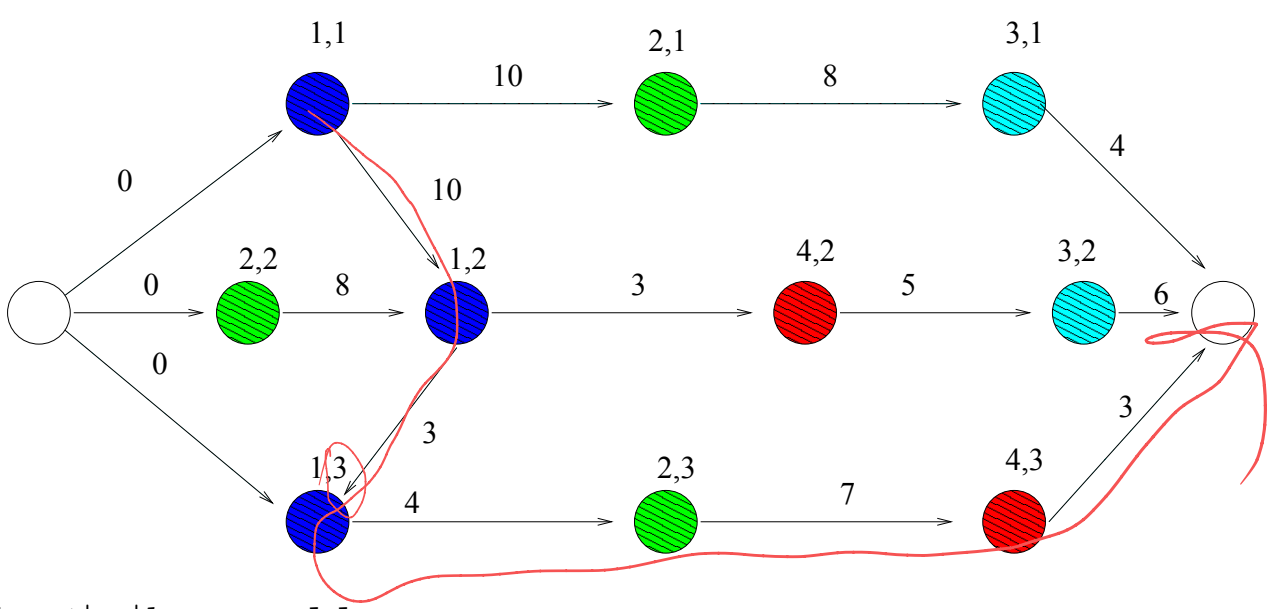
Optimal schedule **2,3,1** $L_{\max}(2) = 5$

Similarly $L_{\max}(3) = 4$. $L_{\max}(4) = 0$.

Schedule M_1 **in the order 1,2,3.**

Cp=27

Iteration 2



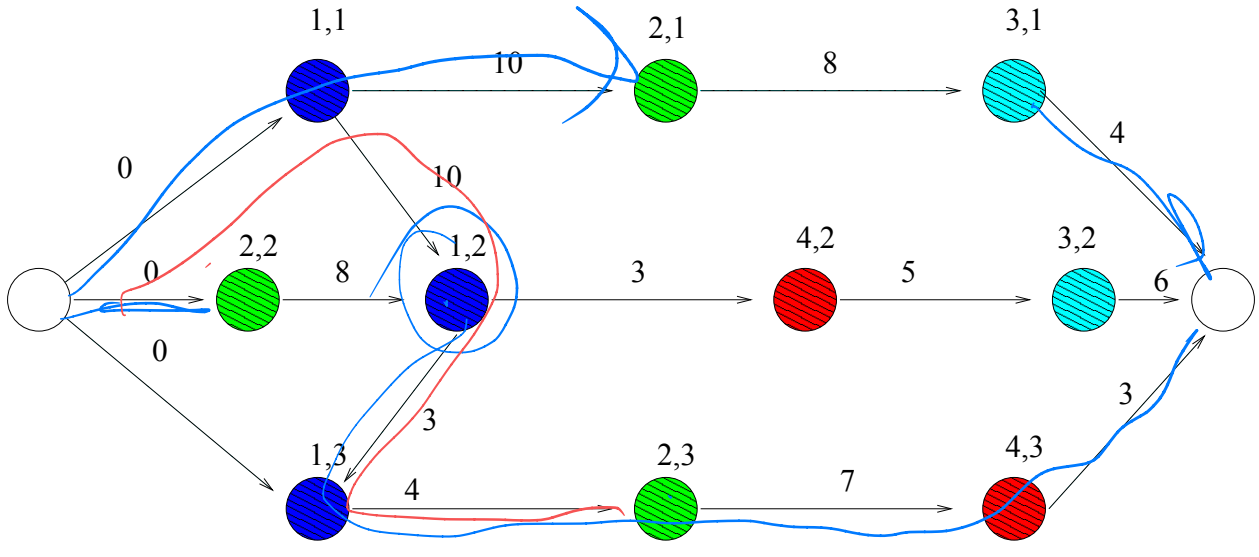
Form the $1|r_j|l_{\max}$ problems

Machine 2

job	**1**	**2**	**3**
r_j	**10**	**0**	**17**
p_j	**8**	**8**	**7**
d_j	**23**	**10**	**14**

Optimal schedule **2,1,3,** $L_{\max}(2) = 1$

Iteration 2



Form the $1|r_j|l_{\max}$ problems

Machine 2

job	1	2	3
r_j	10	0	17
p_j	8	8	7
d_j	23	10	24

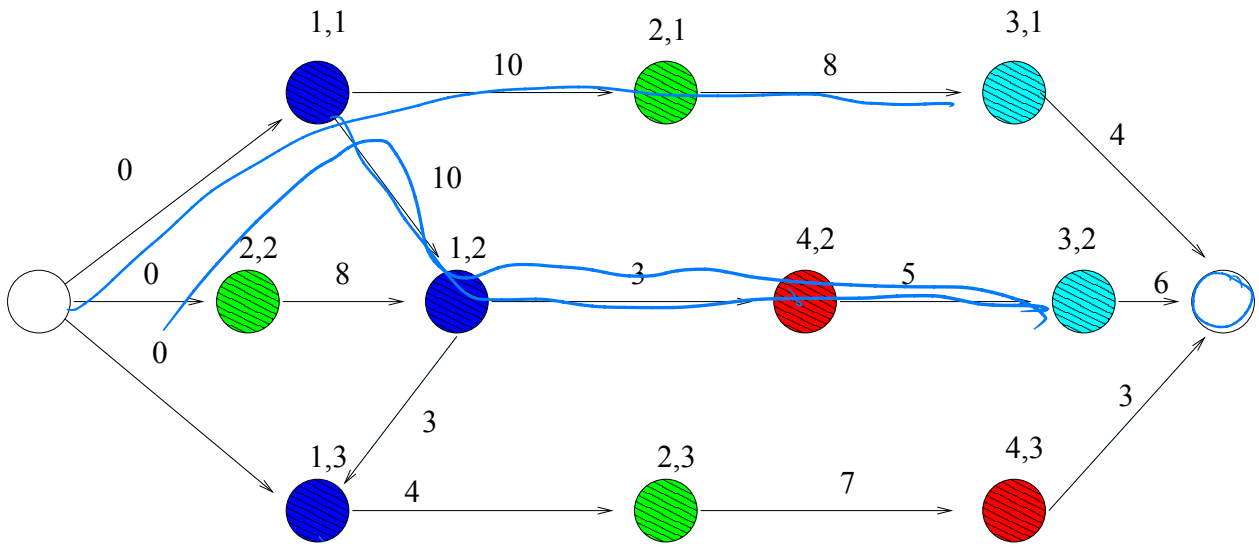
Optimal schedule **2,1,3,** $L_{\max}(2) = 1$

M_2 L_{max}

M_3 1

M_4 0

Iteration 2 (cont)



Form the $1|r_j|L_{\max}$ problems

Machine 3

job	**1**	**2**
r_j	**18**	**18**
p_j	**4**	**6**
d_j	**27**	**27**

Optimal schedule **(either),** $L_{\max}(3) = 1$

Similarly $L_{\max}(4) = 0$**.**

Schedule M_2 **in the order 2,1,3.**

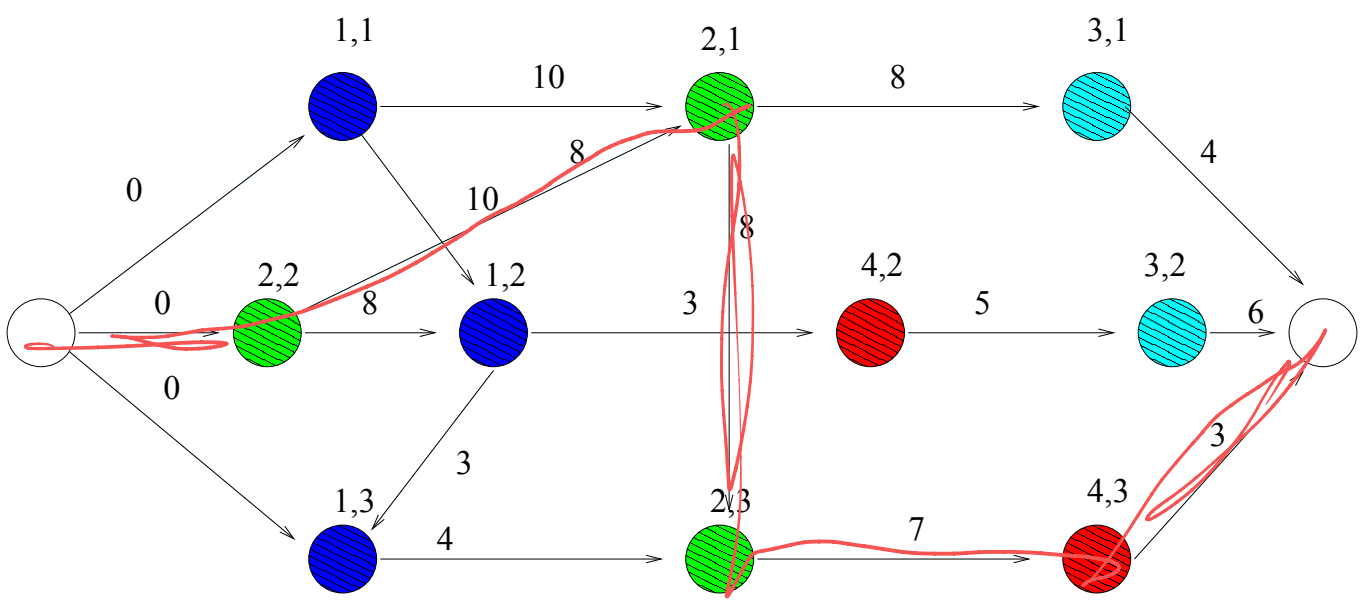
Resequence **Trying to resequence machine 1 does not help.**

Macchè 4

job	2	3
r_j	14	24
p_j	5	0
d_j	21	27

$L_{max} = 0$

Iteration 3



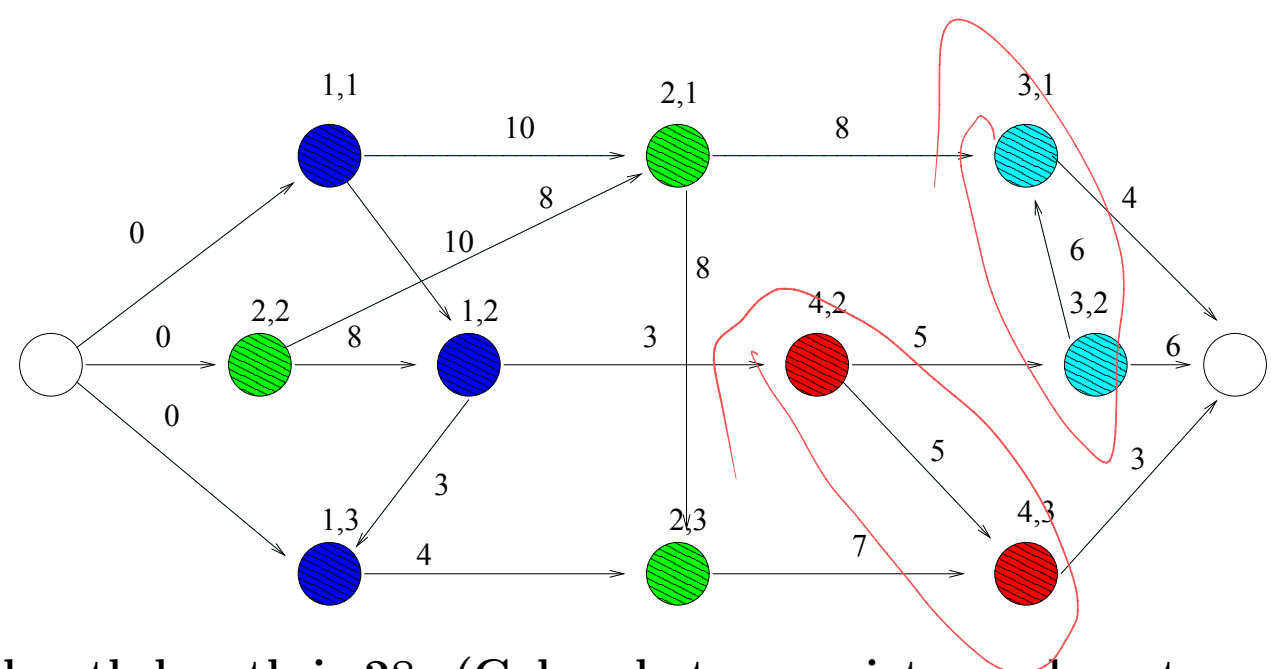
Form the $1|r_j|L_{\max}$ problems
Machine 3 and Machine 4 both have $L_{\max}() = 0$.
Resequencing does not help.

Time: m iterations

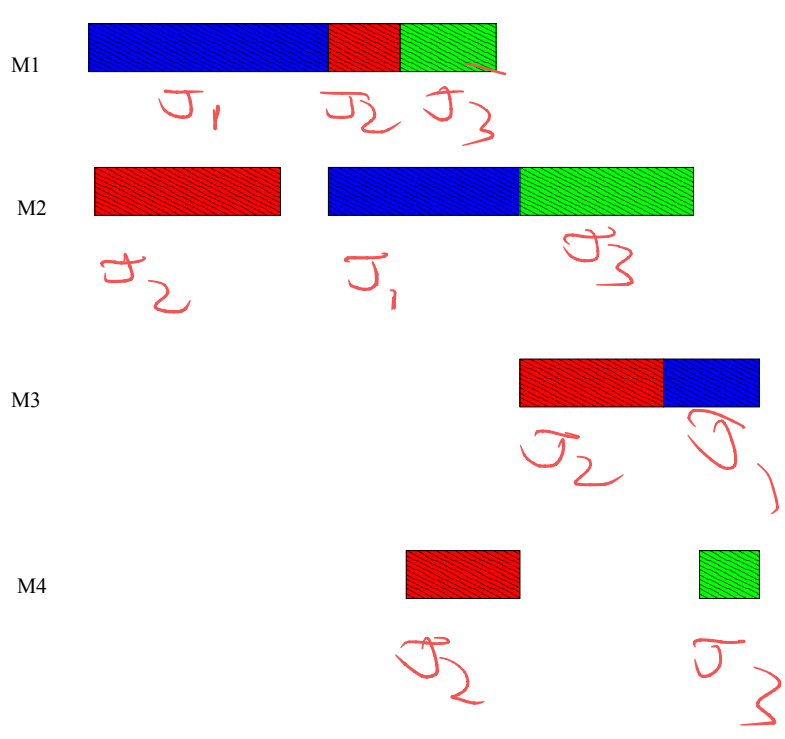
Compute $1|r_j|L_{max}$ for unscheduled nodes ($\leq m$)

Try Rescheduling ($\leq m$)

Final schedule



Critical path length is 28. (Colors between pictures do not correspond)



Solving $O(m^2)$ $1|r_j|L_{max}$ problems with n jobs

Not optimal schedule.